A plasma can be electrically represented as a diode, a resistor, and a capacitor in parallel, as shown in Figure 1. Although this is a very simple model, it represents the basic characteristics of a plasma. The diode effects arise from the fact that the electrons can move much faster than the ions (because the electrons are much lighter). The diode effects can cause a lot of harmonics (multiples of the input frequency) to be generated. These effects are dependent on the process and the chamber, and are of secondary concern when designing a matching network.

Most AC generators are designed to operate into a 50 Ohm load because that is the standard the industry has settled on for measuring and transferring high-frequency electrical power. The function of an impedance matching network, then, is to transform the resistive and capacitive characteristics of the plasma to 50 Ohm, thus matching the load impedance to the AC generator’s impedance.

Two of the most common methods for matching generator and load impedances are:

- Using transformers
- Using matching networks

This paper discusses both methods and also introduces you to the Smith chart, a tool that can be used to calculate the value and type of components needed to construct an impedance matching network.
When we compare the two impedance matching methods mentioned above, we see that they have different applications and strengths:

**Transformer**
- Simplest type of impedance-matching network
- Easy installation
- Simple design
- Low weight
- Inexpensive
- Best used in applications below 1 MHz

Transformers are good for nearly matching the resistive portion of the load. The impedances might not match exactly, so the generators must be able to absorb some reflected power (which usually results in heat).

**Matching Network**
- Used in applications above 1 MHz
- Enables impedances to be matched more exactly than with a transformer
- Generally larger and more difficult to install than a transformer
- More difficult to design than a transformer

Because matching networks do a good job of matching impedances, heating due to reflected power is not typically a problem.

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**Transformers**

Transformers convert source power from one voltage and current level to another voltage and current level. To achieve maximum power transfer from source to load, the load’s impedance must match the characteristic impedance of the generator.

The load impedance is transformed as a square of the voltage-transformation ratio. The ratio of the voltage transformation is referred to as \( N \). \( N \) comes from the number of turns on the input winding divided by the number of turns on the output winding. A transformer with 200 turns on the input and 100 turns on the output is referred to as a 2:1 step-down transformer. If the load is a pure 100 \( \Omega \) resistive load and a transformer with a 2:1 step-down ratio is used (\( N=2 \)), the impedance that the generator “sees” is:

\[
2^2 \times 100 = 400 \, \Omega
\]

**Step Up or Step Down?**

The terms *step up* and *step down* refer to the ability of the transformer to change ("transform") the voltage or the current that passes through it. The amount of power \((V \times I)\) that goes into a transformer is always equal to the amount of power that comes out (discounting negligible losses). This means that while a step-down transformer is changing the input voltage to a lower output voltage, it is also changing the input current to a higher output current. Generally, it is the voltage we are referring to when we talk about *stepping up* or *stepping down.*
If the generator requires a load with a 50 \( \Omega \) impedance and the actual load is 100 \( \Omega \), a transformer with a 1.41:1 step-up voltage ratio is required. Thus, the impedance transformation would be \( 1.41^2 = 2 \), and since this is a step-up transformer, we now divide the load impedance by the transformation ratio to get the input impedance that the generator sees: \( 100/2 = 50 \ \Omega \)

Note that transformer impedance matching only matches the “real” part of the impedance. If there is a large amount of reactance in the load, a transformer will not eliminate these reactive components. In fact, a transformer may exaggerate the reactive portion of the load impedance. This reactive component results in power that is reflected to the generator.

Generators used with impedance matching transformers must be more robust than other types of generators because they must tolerate any reflected power. If the load impedance does not match the characteristic impedance of the generator, power is reflected from the load back to the generator, where it causes higher stresses on the generator components.

**Forward Power, Reflected Power, Load Power**

Forward power is the amount of power that the generator is producing and is trying to send into the load. The reflected power is the amount of power that is “bounced back” from the load. Reflected power results when the load impedance is not matched to the generator’s requirements. Load power is the amount of power that is actually delivered to the load; in most cases, this is accepted as being equal to the forward power minus the reflected power.

Some generators accurately measure the forward and reflected power. When attempting to match the impedances with these generators, the amount of reflected power should be minimized. Depending on the generator, this may be accomplished automatically or by manually selecting the appropriate transformer ratio.

Other generators display the output power, voltage, and current. The mathematical calculation for power is:

\[
P = V \times I \times \cos(\theta)
\]

- \( P = \) power
- \( V = \) voltage
- \( I = \) current
- \( \cos(\theta) = \) cosine of the phase angle between the voltage and current

When using this type of instrumentation, it is critical to optimize the product of the voltage and current so that it comes as close as possible to the actual power that is being displayed. This minimizes the phase angle of the voltage and current, ensuring that the maximum possible amount of energy is being transferred to the load.

**Impedance Matching Networks (Tuners)**

For applications above 1 MHz, an impedance matching network, or tuner, is needed. The primary devices used in constructing an effective impedance matching network are capacitors and inductors.

**Series Elements**

Elements are in series when one end of the subject device is electrically tied to the next device. Current must go through the first device to get to the next device.

**Shunt Elements**

Elements are in parallel (shunt) when both ends of the subject device are electrically tied to the next device. Current reaches both devices at the same time.

In Figure 3, we see that capacitor A is in series with inductor B, and capacitor C is in shunt (parallel) with inductor D.
Conversion Between Elements
The following equations show one way to convert component values between series and shunt circuits:

\[ Rs = \frac{Rp}{1+(Rp/Xp)^2} \]
\[ Xs = \frac{RsRp}{Xp} \]
\[ Rp = \frac{(Rs^2+Xs^2)}{Rs} \]
\[ Xp = \frac{(Rs^2+Xs^2)}{Xs} \]

Rs=series resistance  
Xs=series reactance  
Rp=parallel resistance  
Xp=parallel reactance

Smith Charts
Calculating the values of the devices needed to build an impedance-matching network requires an understanding of the characteristic impedance of the generator and the load impedance. The plasma can be modeled as having an electrical impedance. This facilitates the use of standard electronic matching network design techniques.

In actual design, other factors, such as the power limitations of the components and the effects of the components’ packaging, must also be understood and considered. However, we will focus on impedance during this discussion because it is the fundamental parameter to understand.

One way to determine the device values is to do a numerical analysis of the circuit, which can be time consuming and complex. Another way is to do a graphical analysis using a Smith chart, which is shown in Figure 4.

Smith charts contain many coordinate grids used to calculate electrical characteristics for electronic circuits. In modern use, transparent plastic sheets with Smith charts printed on them allow the designer to lay one chart on another to calculate device values. Many computerized design programs also incorporate Smith charts.

To understand how a Smith chart is constructed, look at Figure 5, which is a basic X-Y coordinate system. We can plot impedance values on this chart in two dimensions.
Figure 6 is similar to Figure 5, but with logarithmic scaling on the X and Y axes.

Now, if we bend around the endpoints (infinity symbols) of the X-Y coordinate system as shown in Figure 7, we see that it forms a Smith chart (Figure 8).

A close examination of Figure 8 reveals the purpose of each line of the chart. The straight line running horizontally represents the real resistance of the circuit. This line has remained unchanged from the original X-Y coordinate system; only the 0 to $+\infty$ is shown. This means that the circles (which are sometimes referred to as constant resistance circles) that cross the center line represent the constant, “real” resistance of the circuit. These are the vertical reference lines on the original graph.

The arcs running adjacent to the center line represent the “imaginary” value of the reactance (capacitance and inductance) in the circuit. These were the horizontal reference lines on the original graph.

Notice that the circle that runs through the center of the chart has a “1” label. This is referred to as the unit circle. When you work with a particular characteristic impedance, for example 50 $\Omega$, all reference points on the chart are multiplied by the value of that impedance, and calculations are made based on the new reference point values.

By examining Figure 9, we see that:

- **Capacitors** have negative impedance values on the impedance chart, they are represented below the center line.
- **Inductors** have positive impedance values, they are represented above the center line.

Essentially, there are two forms of the Smith chart: the impedance chart and the admittance chart, shown in Figure 10.
The admittance chart is a mirror image of the impedance chart. Admittance is the reciprocal of impedance, and it is measured in siemens. Because of the reciprocal relationship of these two charts, the signs for the “j” values are opposite.

Note that the newer term “siemens” has replaced “mhos” as the IEEE-preferred unit of measurement for admittance.

When designing circuits, it is convenient to overlay the impedance chart and the admittance chart as shown in Figure 11. As you can see, when you are working with paper, the overlaid charts are very busy and hard to work with. Sometimes transparencies are used so that the required information can easily be transferred between the impedance and the admittance charts. Some computer programs facilitate the transferring of information easily as well. Later in this paper, we use an impedance chart with just the unit circle from the admittance chart drawn in for clarity.

Admittance, as mentioned in AE® white paper Introducing Power Supplies and Plasma Systems, represents the ease of current flow in a circuit. Admittance = \frac{1}{\text{impedance}}

So, for example, if a circuit has 50 Ω of impedance, its admittance is: Admittance = \frac{1}{50 \, \Omega} = 0.02 \, \text{S}

By using the Smith chart, we can determine what type of device is needed for a particular part of the impedance matching circuit, and what the value of that device must be. We learn this by starting with the load impedance, and translating it on the chart until we reach the optimum impedance point. For our example, the optimum impedance point is 50 Ω because the output circuits of most generators for plasma systems are designed to deliver power into 50 Ω loads.

Impedance and admittance charts are used to calculate the component values needed for devices in different parts of the impedance matching circuit. The impedance chart provides device values for elements placed in series in the circuit, while the admittance chart provides device values for shunt (parallel) elements.

Using an impedance chart implies the use of series devices. Figure 12 shows that, on an impedance chart, moving in a clockwise direction along the real axis tells us that we must add a series inductor. Moving counterclockwise tells us to add a series capacitor.

Figure 12. Moving around the constant resistance circles on an impedance Smith® chart specifies what type of series device to add

Remember that these circular lines were vertical lines on an X-Y coordinate system. Moving up on an X-Y graph is the same as adding inductance, while moving down is the same as adding capacitance. These lines have been converted to circles, so now the movement has been translated in clockwise and counterclockwise rotation.

To illustrate how this occurs, let’s look at two examples. Figure 13 shows point A where the load impedance value is 10-j20 Ω. If we wanted to make this a pure, real resistance of 10 Ω, we could cancel out the -j20 Ω of capacitance with +j20 Ω of inductance.
Figure 13. Example of how adding a series inductor results in clockwise movement of the chart

Figure 14 starts at point B where the load impedance is $10 + j20 \, \Omega$. If we add a series capacitor of $-j20 \, \Omega$, the resulting impedance would be $10 + j0 \, \Omega$.

Figure 14. Example of how adding a series capacitor results in counterclockwise movement of the chart

Using an admittance chart implies the use of shunt devices. Figure 15 shows that, on an admittance chart, moving in a counterclockwise direction along the real axis tells us to add a shunt inductor. Moving clockwise tells us to add a shunt capacitor.

Figure 15. Moving along the real axis on an admittance Smith chart specifies what type of shunt device to add.

Figure 16. Example of how adding a shunt capacitor results in clockwise movement of the chart

Once again, two examples illustrate how this occurs. Figure 16 shows point A where the load admittance is $20 - j40 \, mS$. If we add a capacitor, we move clockwise to the point where we intersect the line that will bring us back to point C ($20 + j0 \, mS$).
Figure 17. Example of how adding a shunt inductor results in counterclockwise movement

Figure 17 starts at point B where the admittance is 20+j40 mS. This time we add an inductor with a value of -j40 mS, which moves us counterclockwise on the Smith chart, resulting in a total admittance of 20+j0 mS.

Using the Smith® Chart

To illustrate how a Smith chart can be used, we’ll design a simple impedance-matching network between a generator and a plasma chamber as shown in Figure 18.

Our goal is to develop a straightforward, simple design for our impedance matching network. We know that the load impedance of the plasma chamber is 10-j30 Ω (which can be obtained by measuring the chamber or the existing impedance matching network). We want to translate that load impedance so that it looks like a pure 50 Ω load to the generator. As illustrated in Figure 19, this is accomplished by using both the impedance and admittance charts.

Figure 18. Simplified diagram of an impedance matching network

Figure 19. You want to make the load impedance of the plasma chamber look like a pure 50 Ω load.
In Figure 20, we start on an impedance chart from point A (10-j30). This chart has the unit circle for an admittance chart dotted in. Ideally, we would show an entire admittance chart overlaying the impedance chart. To minimize confusion, we will just show the unit circle of the admittance chart. The unit circle is important because it travels through the 20 mS point (50 Ω point) on the charts.

We are interested in the distance between the load impedance, point A, and the admittance unit circle. You may note that there are two intersecting points. Both of these points are valid to use; the ensuing calculations would result in a matching network that worked fine for this particular load impedance. However, we will use the farthest point; it is the one that is most commonly used for plasma impedance matching, because it allows for a wider tuning range. This brings us to point B. Because we are moving clockwise on the impedance chart, we know that we must use a series inductor with this reactive value. The total reactance is equal to 50 Ω. At 13.56 MHz, this equates to a 587 nH inductor. Figure 20 also shows the equivalent circuit.

Now, we can switch to the admittance chart. If we were to overlay the two charts (impedance and admittance), we could stick a pin through the top one and find the corresponding equivalent point on the lower chart. How is this possible? Remember that the impedance is a complex number; at point B on the impedance chart, it is 10+j20. The admittance is the reciprocal of the impedance. Take the reciprocal of the impedance, and then multiply by the conjugate over the conjugate (which equals 1) to remove the j from the denominator:

\[
\frac{1}{10+j20} \times \frac{(10-j20)}{10-j20} = \frac{(10-j20)}{(100+j200-j200+400)}
\]

\[
= \frac{(10-j20)}{500} = 0.02 - j0.04 \text{ S} = 20-j40 \text{ mS}
\]

Point B on the impedance chart is the same as point B on the admittance chart; we are just displaying it in different units. Now, we can rotate the point along the unit admittance circle to get to the desired 20 mS (point C). The direction is clockwise, which tells us that we need a shunt capacitor. The distance to point C is 40 mS, which tells us that at a frequency of 13.56 MHz, the value of the capacitor should be 470 pF. Figure 21 shows what the resulting circuit looks like.
This completes the design of a simple, single-point impedance matching network. AE engineers use Smith charts to design impedance-matching networks for various types of plasma systems.

For more information about Smith charts, refer to *Electronic Applications of the Smith Chart* by Phillip H. Smith (ISBN 0-89874-552-7).

Figure 21. Using an admittance chart to translate the susceptance along a constant conductance circle.
white paper